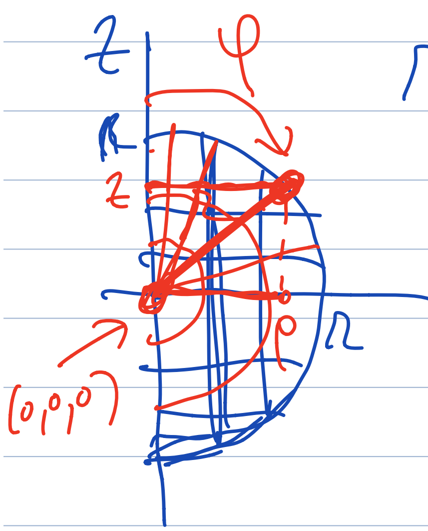
CDI-II — Prática 28/4/21

Ficha 7

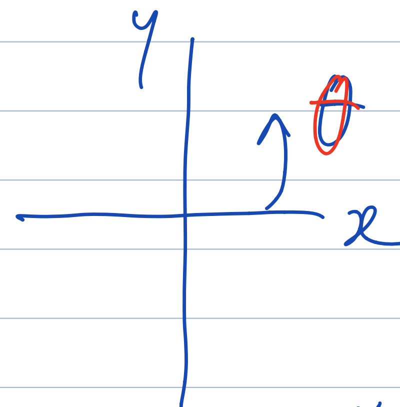
Cilíndricas (ρ, θ, z) $\rho\, d\rho\, d\theta\, dz$

Esféricas $\boxed{(r, \theta, \varphi)}$ $r^2 \operatorname{sen}\varphi\, dr\, d\theta\, d\varphi$



$r^2 = \rho^2 + z^2$

$$\boxed{\begin{aligned} \rho &= r \operatorname{sen}\varphi \\ z &= r \cos\varphi \end{aligned}}$$

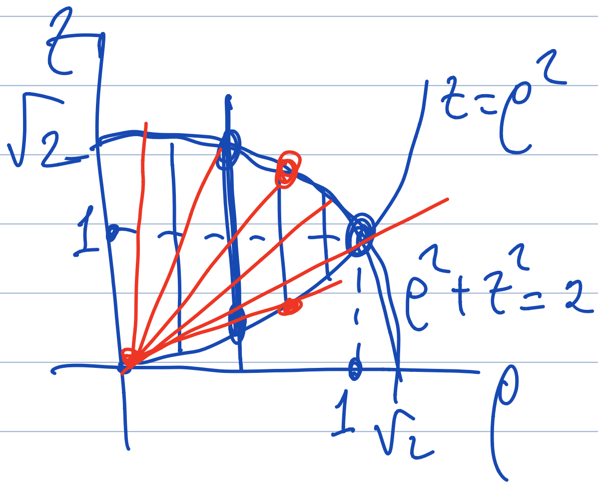


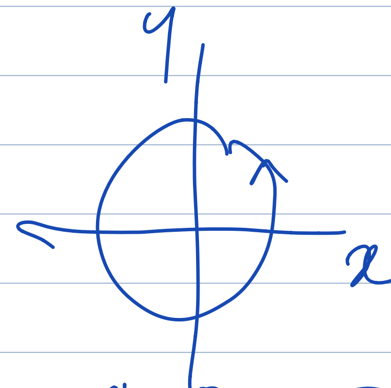
$\theta = \arctan \dfrac{y}{x}$

$r = \sqrt{x^2+y^2+z^2}$ = distância à origem $(0,0,0)$

$\rho = \sqrt{x^2+y^2}$ = distância a Oz

5 - a) $\rho^2 < z < \sqrt{2-\rho^2}$





$0 < \theta < 2\pi$

$z = \sqrt{2-\rho^2}$

$z^2 + \rho^2 = 2$

$d\theta\, dz\, d\rho$ ou

$\underbrace{dz\, d\rho}\, d\theta$

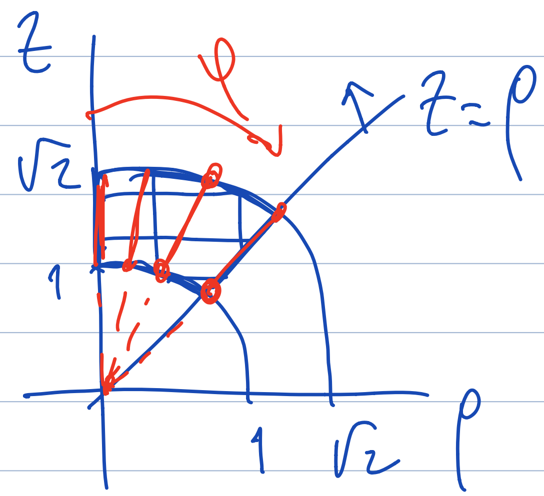
$$\mathrm{vol}_3(V) = \int_0^{2\pi} \left(\int_0^1 \left(\int_{\rho^2}^{\sqrt{2-\rho^2}} \rho\, dz \right) d\rho \right) d\theta$$

etc.

...

~~esféricas~~ (2 integrais)

5-b) $\underbrace{y>0}$, $1<\rho^2+z^2<2$, $z>\rho$



$$\boxed{0<\theta<\pi}$$

$$0<\varphi<\frac{\pi}{4}$$

$$1<r<\sqrt{2}$$

$$\mathrm{Vol}_3(V)=\int_0^{\pi}\left(\int_0^{\frac{\pi}{4}}\left(\int_1^{\sqrt{2}} r^2 \operatorname{sen}\varphi \, dr\right)d\varphi\right)d\theta$$

etc ...

$X \subset \mathbb{R}^n$, $f : \mathbb{R}^n \to \mathbb{R}$,

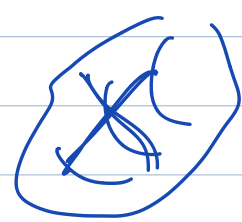
$$\int_X f$$

1- $f \equiv 1 \Rightarrow \displaystyle\int_X f = \int_X 1 = \overset{V}{\overset{\shortparallel}{\mathrm{vol}_n(X)}}$

(<u>volume</u> V)

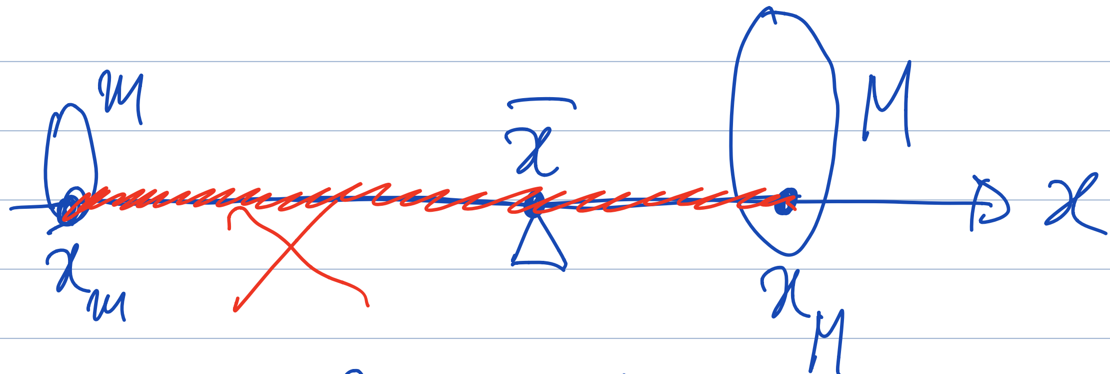
2- <u>Massa</u>

f = densidade de massa: $\sigma = \dfrac{M}{V}$

$\sigma \geq 0$  "sigma"

$$M = \sigma V = \sigma \int_X 1 = \int_X \sigma \quad ✓$$

3 — Centro de massa (centróide)



$$m(\bar{x} - x_m) = M(x_M - \bar{x})$$

$$\bar{x}(m+M) = m x_m + M x_M$$

$$\bar{x} = \frac{m x_m + M x_M}{\boxed{m+M}} \qquad \int_X x\sigma$$

M total do sistema de 2 massas

$$\bar{x} = \frac{\int_X x\sigma}{\int_X \sigma}$$

$$\bar{x} = \frac{\int_X x\,\sigma}{\int_X \sigma}$$

$\bar{y} = \ldots$ $\bar{z} = \ldots$

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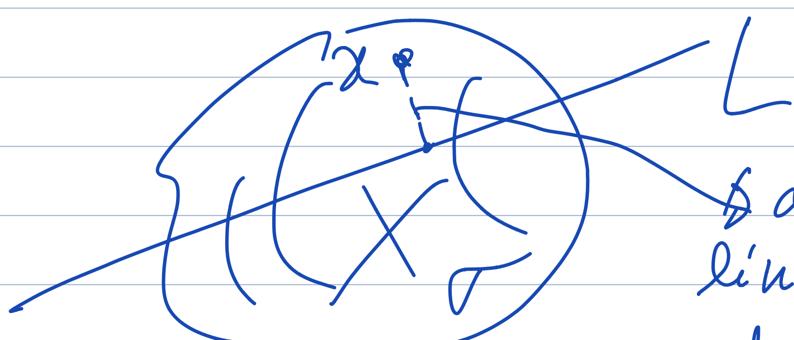
4 — Momento de inércia de X relativo à linha recta L.



distância à linha L.

$d_L(x)$

$$I_L(X) = \int_X \sigma d_L^2$$

Si $L \equiv Oz$ $\quad d_z^2(x,y,z) = x^2+y^2$

— // —

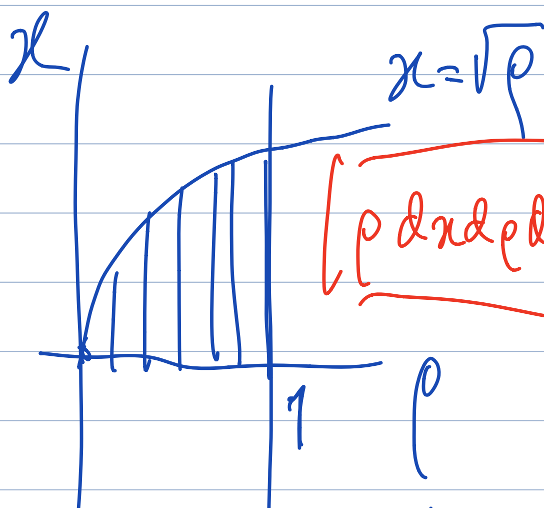
Si $L \equiv Ox$, $d_x^2(x,y,z) = y^2+z^2$

6 - $\nu(x,y,z) = x(y^2+z^2)$

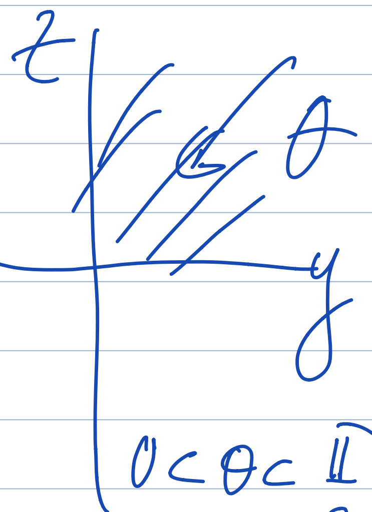
$L = Ox$ $\qquad d_x^2(x,y,z) = y^2+z^2$

U : $\rho^2 < 1$; $0 < x < \rho^{2/4}$, $y>0$, $z>0$

(ρ,θ,x) $\qquad \rho = \sqrt{y^2+z^2}$



$\rho\, dx\, d\rho\, d\theta$



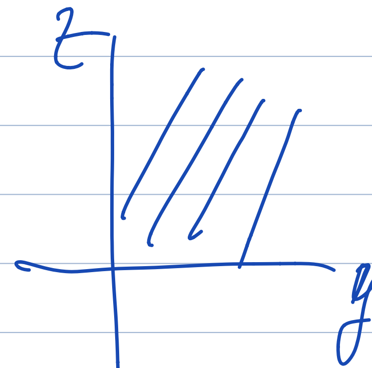
$0<\theta<\frac{\pi}{2}$

$$I_x(U) = \int_0^{\pi/2}\left(\int_0^1\left(\int_0^{\sqrt{\rho}} \rho\, \underbrace{x\rho^2}\, \underbrace{\rho^2}\, dx\right) d\rho\right) d\theta$$

etc...

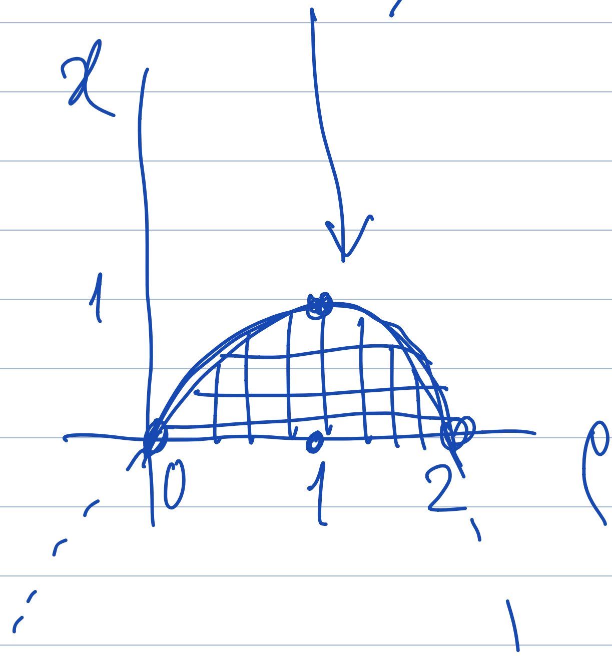
7-a) (ρ,θ,x) $\quad \rho=\sqrt{y^2+z^2}$

$0<x<1-(\rho-1)^2$

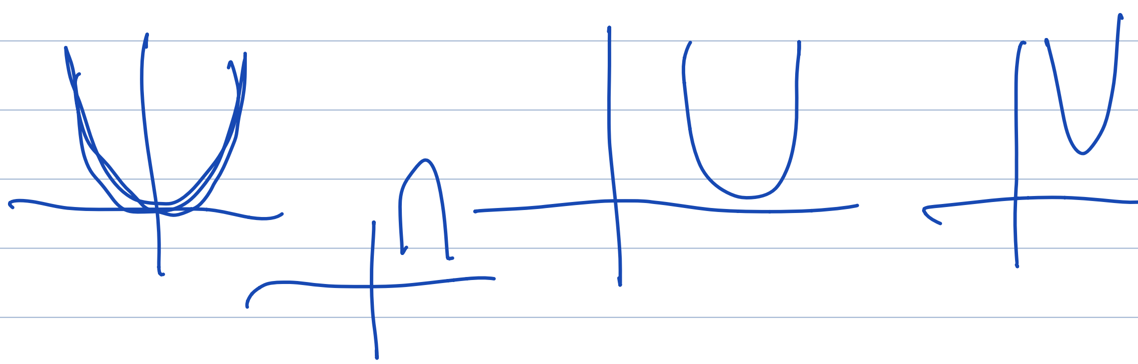


$0<\theta<\frac{\pi}{2}$

$x=1-(\rho-1)^2$

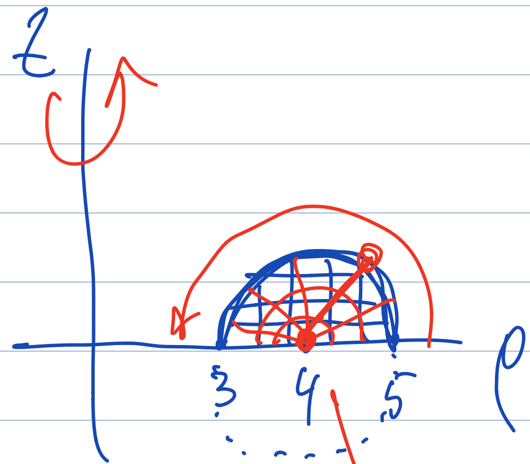


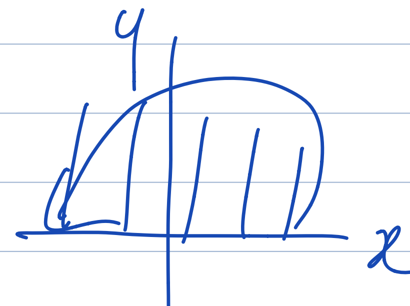
etc...



7-b) $(\rho-4)^2+z^2<1 \; ; \; y\geq 0$

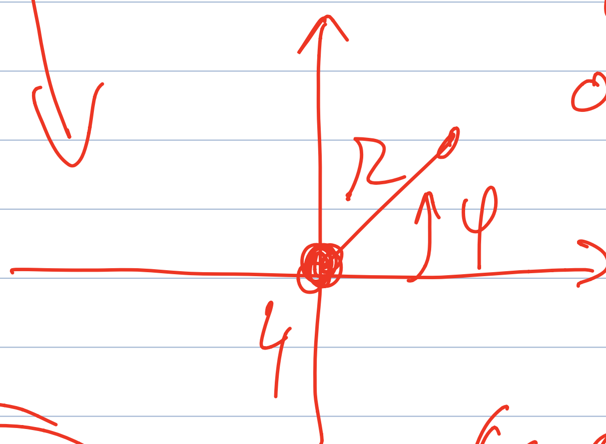
$z\geq 0$





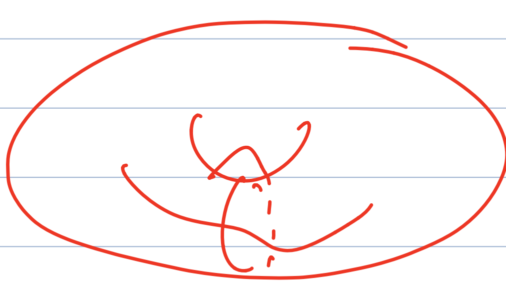
$0<\theta<\pi$

$$\begin{cases} \rho-4 = r\cos\varphi \\ z = r\sin\varphi \end{cases}$$



$0<\varphi<\pi$

$0<r<1$



(r,θ,φ)

Integral e derivada.

CDI-I : TFC

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b(x))\,b'(x) - f(a(x))\,a'(x)$$

CDI-II : Leibnitz

$$F(x) = \int_a^b f(x,t)\,dt$$

$$F'(x) = \frac{d}{dx}\int_a^b f(x,t)\,dt =$$

$$= \int_a^b \frac{\partial f}{\partial x}(x,t)\,dt$$

$$\boxed{\frac{d}{dx}\int_a^b f(x,t)\,dt = \int_a^b \frac{\partial f}{\partial x}(x,t)\,dt}$$

8 – $$F'(t) = \int_0^1 x^2 \cos(tx^2 + x^3)\,dx$$

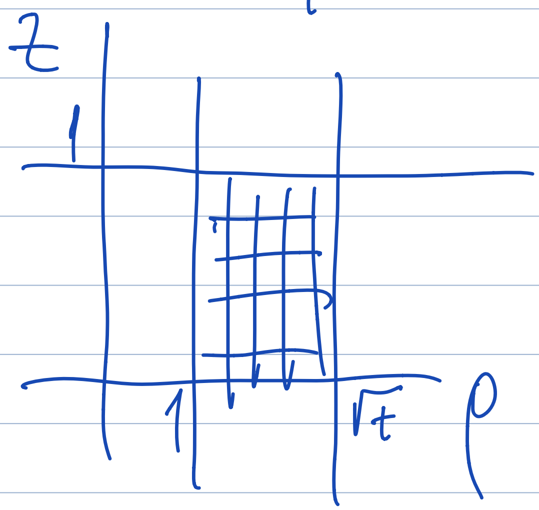
$$F'(0) = \int_0^1 x^2 \cos(x^3)\,dx$$

etc. . .

9 — $1 < \rho^2 < t$

$0 < z < 1$

$y > 0$



$$\boxed{0 < \theta < \pi}$$

$$F(t) = \int_1^{\sqrt{t}} \left(\boxed{\int_0^{\pi} \left(\int_0^1 \rho \frac{e^{t\rho^2}}{\rho^2} dz \right) d\theta} \right) d\rho$$

$$F(t) = \pi \int_1^{\sqrt{t}} \frac{e^{t\rho^2}}{\rho} d\rho$$

$$F(t) = \pi \int_1^{\sqrt{t}} \frac{e^{t\rho^2}}{\rho} \, d\rho$$

$$\sqrt{t} = b(t)$$

$$F(t) = \pi \int_1^{\sqrt{t}} f(t,\rho) \, d\rho$$

$$F(t) = I\big(b(t), t\big)$$

$$F'(t) = \underbrace{\frac{\partial I}{\partial b} b'(t)}_{\text{TFC}} + \underbrace{\frac{\partial I}{\partial t}}_{\text{Leibniz.}}$$

$$F'(t) = \pi \frac{e^{tt}}{\sqrt{t}} \frac{1}{2\sqrt{t}} + \pi \int_1^{\sqrt{t}} \rho^2 \frac{e^{t\rho^2}}{\rho} \, d\rho$$

etc...

$$F'(t) = \pi \frac{e^{t^2}}{2t} + \frac{\pi}{2t} \int_1^{\sqrt{t}} 2t \rho \, e^{t\rho^2} d\rho$$

etc. . . .